



Transportation Science

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Dynamic Facility Location with Generalized Modular Capacities

Sanjay Dominik Jena, Jean-François Cordeau, Bernard Gendron

To cite this article:

Sanjay Dominik Jena, Jean-François Cordeau, Bernard Gendron (2015) Dynamic Facility Location with Generalized Modular Capacities. *Transportation Science* 49(3):484-499. <https://doi.org/10.1287/trsc.2014.0575>

Full terms and conditions of use: <http://pubsonline.informs.org/page/terms-and-conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2015, INFORMS

Please scroll down for article—it is on subsequent pages



INFORMS is the largest professional society in the world for professionals in the fields of operations research, management science, and analytics.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Dynamic Facility Location with Generalized Modular Capacities

Sanjay Dominik Jena

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport (CIRRELT); and
Département d'informatique et de recherche opérationnelle, Université de Montréal, Montréal, Québec H3T 1J4, Canada,
sanjay.jena@cirrelt.ca

Jean-François Cordeau

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport (CIRRELT); and
Canada Research Chair in Logistics and Transportation, HEC Montréal, Montréal, Québec H3T 2A7, Canada,
jean-francois.cordeau@hec.ca

Bernard Gendron

Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport (CIRRELT); and
Département d'informatique et de recherche opérationnelle, Université de Montréal, Montréal, Québec H3T 1J4, Canada,
bernard.gendron@cirrelt.ca

Location decisions are frequently subject to dynamic aspects such as changes in customer demand. Often, flexibility regarding the geographic location of facilities, as well as their capacities, is the only solution to such issues. Even when demand can be forecast, finding the optimal schedule for the deployment and dynamic adjustment of capacities remains a challenge, especially when the cost structure for these adjustments is complex. In this paper, we introduce a unifying model that generalizes existing formulations for several dynamic facility location problems and provides stronger linear programming relaxations than the specialized formulations. In addition, the model can address facility location problems where the costs for capacity changes are defined for all pairs of capacity levels. To the best of our knowledge, this problem has not been addressed in the literature. We apply our model to special cases of the problem with capacity expansion and reduction or temporary facility closing and reopening. We prove dominance relationships between our formulation and existing models for the special cases. Computational experiments on a large set of randomly generated instances with up to 100 facility locations and 1,000 customers show that our model can obtain optimal solutions in shorter computing times than the existing specialized formulations.

Keywords: mixed-integer programming; facility location; modular capacities

History: Received: March 2013; revisions received: October 2013, March 2014, July 2014; accepted: August 2014.
Published online in *Articles in Advance* February 27, 2015.

1. Introduction

Dynamic facility location is about deciding *where* and *when* to provide capacity to satisfy customer demand at the lowest cost. This demand is rarely stable; rather, it increases, decreases, or oscillates over time. Therefore, facility capacities often have to be adjusted dynamically. Many variants of dynamic facility location problems have been studied, suggesting different ways to adjust capacities throughout a given planning horizon. The most common features include capacity expansion and reduction (Luss 1982; Jacobsen 1990; Peeters and Antunes 2001; Troncoso and Garrido 2005; Dias, Captivo, and Clímaco 2007), temporary facility closing (Chardaire, Sutter, and Costa 1996; Canel et al. 2001; Dias, Captivo, and Clímaco 2006), and the relocation of capacities (Melo, Nickel, and Saldanha-da-Gama 2006). Mathematical models that include such features have been applied in both

the private and public sectors to determine locations and capacities for production facilities, schools, hospitals, libraries, and many other facilities.

Facility location decisions aim to strike a balance between the fixed costs to supply capacity and the allocation costs to serve the demand. The latter often correspond to transportation costs to deliver products or provide services to customers. The ratio between these two types of costs has a strong impact on the solution and the difficulty of solving the problem (see, e.g., Shulman 1991; Melkote and Daskin 2001). In dynamic facility location problems, a detailed representation of the transportation costs affects not only the facility locations but also their capacity throughout the planning horizon, as capacity tends to follow the demand along time.

Regarding the fixed costs to provide the capacity, many studies acknowledge the existence of economies

of scale (Correia and Captivo 2003; Correia, Gouveia, and Saldanha-da-Gama 2010). Previous work considered economies of scale mainly for the construction and production costs, but the costs for adjusting the capacities of the facilities have commonly been modeled in less detail. However, the latter is necessary to ensure a fair representation of the cost structure found in practice. The costs to adjust capacities often depend not only on the size of the adjustment but also on the current capacity level. This is true in a large class of applications, especially in transportation, logistics, and telecommunications, where additional capacity gets cheaper (or more expensive) when approaching the maximum capacity limit.

In this work, we introduce a very general dynamic facility location problem, referred to as the *Dynamic Facility Location Problem with Generalized Modular Capacities* (DFLPG). The problem allows modular capacity changes subject to a detailed cost structure and is modeled as a mixed-integer programming (MIP) formulation. Because of its generality, this model unifies several existing problems in the literature. The cost structure used in the model is based on a matrix describing the costs for capacity changes between all pairs of capacity levels. We are not aware of any other work dealing with facility location with a similar level of detail in the cost structure.

Our study is motivated by an industrial project with a Canadian logging company that must locate camps for workers involved in wood harvest activities while optimizing the overall logistics and transportation costs (Jena, Cordeau, and Gendron 2012). In this problem, the total capacity of a camp is represented by its number of hosting units, and additional units provide supporting infrastructure. As the relation between the number of different units is nonlinear, the costs for capacity changes are described in a transition matrix.

The contribution of this work is threefold. First, we introduce a general dynamic facility location model that comprises a large set of existing formulations. Second, we analyze the linear programming (LP) relaxation bound obtained by our model, showing that it is at least as strong as the LP relaxation bound of existing specialized formulations. Third, we perform a computational study on a large set of randomly generated instances, showing that our model, when solved with a state-of-the-art MIP solver, can obtain optimal solutions in shorter computation times than the specialized formulations.

The paper is organized as follows. In §2, we present a survey of the relevant literature. Then, §3 introduces a linear MIP formulation for the DFLPG and shows how this model can be used to represent two important special cases. To compare the resulting models with alternative formulations, §4 derives specialized

formulations for the two special cases, based on existing models from the literature. We identify a weak point in one of the existing formulations and suggest a set of valid inequalities to make it as strong as our model. Dominance relations are proved between all formulations, showing that our model is at least as strong as each of the specialized formulations. The presented models are then compared by means of computational experiments in §5. Conclusions follow in §6.

2. Literature Review

Most dynamic facility location problems can be seen as multiperiodic extensions of classical location problems, such as the *Capacitated Facility Location Problem* (CFLP). However, dynamic facility location problems commonly involve further extensions. As pointed out by Arabani and Farahani (2011), the notion of what dynamic means may differ when dealing with different areas of facility location. Its definition thus strongly depends on the application context. For example, school capacities may be increased or decreased to meet demographic trends (e.g., Peeters and Antunes 2001), terminals in telecommunication networks may be installed and removed to adapt to changes in data traffic and costs (e.g., Chardaire, Sutter, and Costa 1996), and hospitals may relocate ambulances to cope with unpredictable demand (e.g., Brotcorne, Laporte, and Semet 2003). Owen and Daskin (1998) review works that treat either dynamic or stochastic facility location problems. Farahani, Abedian, and Sharahi (2009) deal with dynamic aspects of facility location problems and propose several classification criteria. A book chapter by Jacobsen (1990) dedicated to multiperiod capacitated location models thoroughly discusses models that allow capacity expansion. Luss (1982) focuses on capacity expansion and reviews the literature and applications in the context of problems with a single, two, and multiple facilities. Although not explicitly focusing on dynamic aspects, many other works introduced classifications for location problems that often also apply to features that can be found in dynamic location problems. These include, among many other studies, the works of Hamacher and Nickel (1998), Owen and Daskin (1998), Klose and Drexler (2005), Daskin (2008), and Melo, Nickel, and Saldanha-da-Gama (2009).

The choice of the facility type or size has also been considered in several works. In particular, Shulman (1991), Correia and Captivo (2003), and Troncoso and Garrido (2005) consider such choice, which implies different capacities and costs for each facility type. The last authors apply the model to the forestry sector, where facilities of different sizes may also be expanded. Dias, Captivo, and Clímaco (2007) focus

on modular capacity expansion and reduction. Wu, Zhang, and Zhang (2006) present a facility location problem in which the facility setup costs depend on the number of facilities placed at a site. To represent economies of scale, all of the cited works use binary variables to distinguish different facility sizes. Capacity level changes consider only the amount of capacity added or removed. However, the previous capacity level is not taken into consideration. Some authors such as Harkness (2003) also recognize the importance of inverse economies of scale, where the unit price increases as the facility gets larger.

To dynamically adjust capacity to demand changes, the best choice depends on the demand forecast and the costs involved in capacity changes. For example, if capacity is leased, it may be possible to terminate a leasing contract at any time. In other situations, it may be beneficial to temporarily close a facility to avoid high maintenance costs. This may be appropriate when demand temporarily decreases, but it is likely to return to its previous level afterward. The closing and reopening of facilities may be partial or complete. Previous studies focused mostly on temporarily closing entire facilities. Among the suggested models, some are limited to a single closing and reopening of each facility, whereas others allow repeated closings and reopenings throughout the planning horizon. The uncapacitated facility location problem presented by Van Roy and Erlenkotter (1982) and the supply chain model of Hinojosa et al. (2008) allow one-time opening or closing of facilities: new facilities can be opened once and existing facilities can be closed once. Chardaire, Sutter, and Costa (1996) and Canel et al. (2001) propose formulations for opening and closing facilities more than once. Both works use binary variables to represent the state of the facility. The objective function contains a bilinear term to represent a state change from open to closed or vice versa. A linear formulation for a simplified version of this problem, treating only a single capacity level, has been proposed by Dias, Captivo, and Clímaco (2006). Binary variables with two time indices indicate the period throughout which a facility is open. The cited works interpret facility closing either as temporary (i.e., the facility still exists, but its capacities are temporarily unavailable) or permanent. In most cases, maintenance costs for temporarily

closed facilities are low and can therefore be ignored in the model. Most of the existing formulations therefore do not explicitly distinguish temporary and permanent facility closing.

When the customer demand permanently changes in a certain region and is not likely to return to its previous level, one may want to expand or reduce the facility capacities to permanently adjust to these new conditions. Luss (1982) observes that models for capacity expansion can be classified into two categories: capacity expansion at a single facility and capacity expansion via a finite set of projects, each holding a certain capacity. The first category includes models that allow one facility at a location and increases or decreases of the available capacity along time. The second category consists of models where multiple facilities are allowed in the same location, each specified by a time interval (a capacity block) of production availability. Figure 1 illustrates both classes. The first class is shown in panel (a), where capacities at the same facility are either increased or decreased. The second class is illustrated by panels (b) and (c), representing two extreme configurations of the capacity blocks. Any configuration between these two is also feasible for the second class.

Models in the first category include those of Melo, Nickel, and Saldanha-da-Gama (2006) and Behmardi and Lee (2008). Both works model capacity expansion and reduction by relocating capacity from or to a dummy location. The authors of the former work model capacities as a continuous flow, but demonstrate how to link the flow to binary variables to restrict capacity changes to modular sizes. Models in the second category do not allow the capacity modification of a facility once it is constructed. However, they allow multiple facilities of different sizes (capacity blocks) at the same location, which is equivalent to the adjustment of the total capacity sum along time. Examples for this class include the works of Shulman (1991), Troncoso and Garrido (2005), and Dias, Captivo, and Clímaco (2007). More restricted types of capacity expansion or reduction have also been presented. In the work of Peeters and Antunes (2001), either a facility expands or decreases

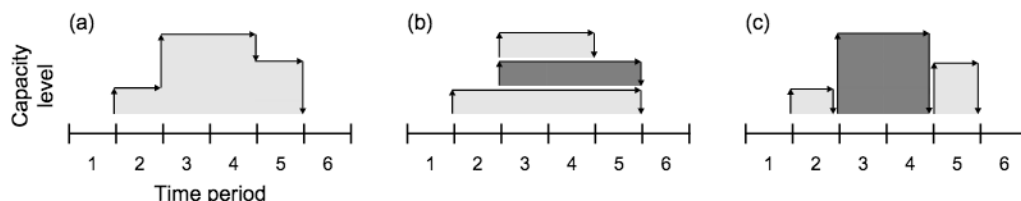


Figure 1 Capacity Expansion/Reduction by Use of a Single Facility (a), Horizontal Capacity Blocks (b), and Vertical Capacity Blocks (c)

its capacity throughout the entire planning horizon. Capacity expansion and reduction at the same location is thus not allowed.

3. Mathematical Formulation

In this section, we give a more formal description of the DFLPG and introduce a MIP model for the problem. We also explain how the different cases described in §2 can be modeled as a DFLPG.

3.1. DFLPG Formulation

We denote by J the set of potential facility locations and by $L = \{0, 1, 2, \dots, q\}$ the set of possible capacity levels for each facility. We also denote by I the set of customer demand points and by $T = \{1, 2, \dots, |T|\}$ the set of time periods in the planning horizon. We assume throughout that the beginning of period $t + 1$ corresponds to the end of period t .

The demand of customer i in period t is denoted by d_{it} . The cost to serve one unit from facility j operating at capacity level l to customer i during period t is denoted by g_{ijlt} . This term is typically a cost function for handling and transportation costs, based on the distance between customer i and facility j . The capacity of a facility of size l at location j is given by u_{jl} (with $u_{j0} = 0$). The cost matrix $f_{jl_1l_2t}$ describes the combined cost to change the capacity level of a facility at location j from l_1 to l_2 at the beginning of period t and to operate the facility at capacity level l_2 throughout the period. Furthermore, we let l^j be the capacity level of an existing facility at location j . The constant l^j is 0 if location j does not possess an existing facility at the beginning of the planning horizon.

To formulate the problem, we use binary variables $y_{jl_1l_2t}$ equal to 1 if and only if the facility at location j changes its capacity level from l_1 to l_2 at the beginning of period t . The allocation variables x_{ijlt} denote the fraction of the demand of customer i in period t that is served from a facility of size l located at j . Based on these definitions, we define the following MIP model, referred to as the *Generalized Modular Capacities* (GMC) formulation:

$$\text{(GMC) } \min \left\{ \sum_{j \in J} \sum_{l_1 \in L} \sum_{l_2 \in L} \sum_{t \in T} f_{jl_1l_2t} y_{jl_1l_2t} + \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} g_{ijlt} d_{it} x_{ijlt} \right\} \quad (1)$$

$$\text{s.t. } \sum_{j \in J} \sum_{l \in L} x_{ijlt} = 1 \quad \forall i \in I, \forall t \in T, \quad (2)$$

$$\sum_{i \in I} d_{it} x_{ijlt} \leq \sum_{l_1 \in L} u_{jl_1} y_{jl_1l_2t} \quad \forall j \in J, \forall l \in L, \forall t \in T, \quad (3)$$

$$\sum_{l_1 \in L} y_{jl_1l(t-1)} = \sum_{l_2 \in L} y_{jl_2t} \quad \forall j \in J, \forall l \in L, \forall t \in T \setminus \{1\}, \quad (4)$$

$$\sum_{l_2 \in L} y_{jl_2t} = 1 \quad \forall j \in J, \quad (5)$$

$$x_{ijlt} \geq 0 \quad \forall i \in I, \forall j \in J, \forall l \in L, \forall t \in T, \quad (6)$$

$$y_{jl_1l_2t} \in \{0, 1\} \quad \forall j \in J, \forall l_1 \in L, \forall l_2 \in L, \forall t \in T. \quad (7)$$

The objective function (1) minimizes the total cost for changing the capacity levels and allocating the demand. Constraints (2) are the demand constraints for the customers. Constraints (3) are the capacity constraints at the facilities. Constraints (4) link the capacity change variables in consecutive time periods. Finally, constraints (5) specify that exactly one capacity level must be chosen at the beginning of the planning horizon. Note that the flow constraints (4) further guarantee that, at each time period, exactly one capacity change variable is selected.

Valid Inequalities. To facilitate the solution of the GMC, we may additionally use two types of valid inequalities. The *Strong Inequalities* (SI) used in facility location and network design problems (see, e.g., Gendron and Crainic 1994) are known to provide a tight upper bound for the demand assignment variables. These inequalities can be adapted to our model as follows:

$$x_{ijlt} \leq \sum_{l_1 \in L} y_{jl_1t} \quad \forall i \in I, \forall j \in J, \forall l \in L, \forall t \in T. \quad (8)$$

The SIs may be added to the model either a priori or in a branch-and-cut manner only when they are violated in the solution of the LP relaxation. The second set of valid inequalities is referred to as the *Aggregated Demand Constraints* (ADC). Although they are redundant for the LP relaxation, adding them to the model enables MIP solvers to generate cover cuts that further strengthen the formulation

$$\sum_{j \in J} \sum_{l_1 \in L} \sum_{l_2 \in L} u_{jl_2} y_{jl_1l_2t} \geq \sum_{i \in I} d_{it} \quad \forall t \in T. \quad (9)$$

3.2. DFLPG-Based Models for the Special Cases

We now explain how two important special cases can be modeled with the GMC formulation: first, *facility closing and reopening* and, second, *capacity expansion and reduction*.

The first problem considered here allows the construction of at most one facility per location. The size of the facility is chosen from a discrete set of capacity levels. Existing facilities may be closed and reopened multiple times. Note that, in this problem, facility closing does not refer to permanent closing, but only to

the temporary closing of a facility. We therefore distinguish costs for the construction of a facility, for temporarily closing an open facility, for reopening a closed facility, and for maintaining open facilities. As in most of the previous literature, we do not consider maintenance costs for temporarily closed facilities. We denote this problem as the *Dynamic Modular Capacitated Facility Location Problem with Closing and Reopening* (DMCFLP_CR).

In the second problem, capacities can be adjusted by the use of a single facility at each location. At each facility, the capacity can be expanded or reduced from one capacity level to another. We assume that an expansion of l capacity levels always has the same costs, regardless of the previous capacity level. We assume the same for the reduction of capacities. We denote this problem as the *Dynamic Modular Capacitated Facility Location Problem with Capacity Expansion and Reduction* (DMCFLP_ER).

In addition to the input data already defined for the DFLPG, we define the following fixed costs to characterize these two special cases:

- c_{jl}^c and c_{jl}^o are the costs to temporarily close and reopen a facility of size l at location j , respectively.
- f_{jl}^c and f_{jl}^o are the costs to reduce and expand the capacity of a facility at location j by l capacity levels, respectively.
- F_{jl}^o is the cost to maintain an open facility of size l at location j throughout one time period.

For the sake of simplicity and without loss of generality, we assume that all of these costs do not change during the planning horizon.

In the GMC, capacity level changes are represented by the $y_{j\bar{l}_1\bar{l}_2t}$ variables. These transitions from one capacity level to another can be represented in a graph, where each node represents a capacity level and each arc a capacity level transition. To model the special cases, we choose a certain subset of arcs, as well as their corresponding objective function coefficients $f_{j\bar{l}_1\bar{l}_2t}$. Note that, although the costs for the GMC can be based on a cost matrix, the costs for the special cases are based on a cost vector. The cost coefficients $f_{j\bar{l}_1\bar{l}_2t}$ correspond to combinations of different operations, for example, the cost to expand capacity plus the maintenance costs for the new capacity level.

For the problem variant involving facility closing and reopening, we create an artificial capacity level \bar{l} for each capacity level $l \in L \setminus \{0\}$. Capacity level \bar{l} represents the state in which a facility of size l is temporarily closed. At each time period $t \in T$ and location $j \in J$, we may find different arc types $y_{j\bar{l}_1\bar{l}_2t}$ to model capacity level changes (note that the cost for an arc is usually composed of the cost to perform the capacity transition and the maintenance costs for the new capacity level):

1. Facility construction and capacity expansion. The expansion of the capacity is represented by

an arc from capacity level l_1 to any other capacity level $l_2 > l_1$. If the arc represents a facility construction, then l_1 is 0. The capacity is thus expanded by $l_2 - l_1$ capacity levels. The cost for this arc is set to $f_{j\bar{l}_1\bar{l}_2t} = f_{j(l_2-l_1)}^o + F_{j\bar{l}_2}^o$.

2. Capacity reduction. The reduction of the capacity is represented by an arc from capacity level l_1 to any other capacity level $l_2 < l_1$. The capacity is thus reduced by $l_1 - l_2$ capacity levels. The cost for this arc is set to $f_{j\bar{l}_1\bar{l}_2t} = f_{j(l_1-l_2)}^c + F_{j\bar{l}_2}^o$.

3. Maintaining the current capacity level. A facility may neither expand nor reduce the current capacity level. The cost of this arc is thus only composed of the maintenance cost, that is, $f_{j\bar{l}_1\bar{l}_1t} = F_{j\bar{l}_1}^o$ if the capacity level represents an open facility, $f_{j\bar{l}_1\bar{l}_1t} = 0$ if the capacity level represents a temporarily closed facility, and $f_{j00t} = 0$ if no facility exists.

4. Temporary closing. An open facility of size l_1 can be temporarily closed; that is, it changes to capacity level \bar{l}_1 . The total cost is $f_{j\bar{l}_1\bar{l}_1t} = c_{j\bar{l}_1}^c$.

5. Reopening a closed facility. A temporarily closed facility of size l_1 can be reopened; that is, it changes its capacity level from \bar{l}_1 to l_1 . The total cost for this arc is $f_{j\bar{l}_1\bar{l}_1t} = c_{j\bar{l}_1}^o + F_{j\bar{l}_1}^o$.

The DMCFLP_CR is represented by arcs of types 1 (for construction only), 3, 4, and 5. We denote the resulting model as the CR-GMC formulation. The DMCFLP_ER is represented by arcs of types 1, 2, and 3. The resulting model is denoted as the ER-GMC formulation.

4. Comparisons with Specialized Formulations

We now present alternative formulations for the two special cases. These formulations are adaptations of existing models proposed in the literature. For each problem, we present formulations based on two different modeling approaches, as presented in §2: location variables with one time index and location variables with two time indices.

4.1. Facility Closing and Reopening

We consider models for the problem DMCFLP_CR.

4.1.1. Single Time Index Flow Formulation. This model can be seen as an extension of existing dynamic facility location problems (Shulman 1991). Flow conservation constraints such as those used in the relocation model of Wesolowsky and Truscott (1975) are adapted to model facility closing and reopening. The model is based on the following variables. The demand allocation from facilities to customers is given by x_{ijtt} . Binary variable s_{jtt} is 1 if a facility of size l is constructed at the beginning of period t at location j ; binary flow variable y_{jlt} indicates whether a facility of size l is available at location j during time

period t . Finally, binary variables v_{jlt}^o and v_{jlt}^c are equal to 1 if a temporarily closed facility at location j of size l is reopened at the beginning of period t and if an open facility at location j of size l is temporarily closed at the beginning of period t , respectively. The input data are as defined in §3.2. Note that certain equations may include terms that are not defined for a certain variable index; for example, index $(t - 1)$ is not defined for $t = 1$. Undefined terms are assumed to take the value 0. The *single time index flow formulation* (CR-1I) is given by

$$(CR-1I) \min \left\{ \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} (f_{jl}^o s_{jlt} + F_{jl}^o y_{jlt} + c_{jl}^o v_{jlt}^o + c_{jl}^c v_{jlt}^c) + \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} g_{ijlt} d_{it} x_{ijlt} \right\} \quad (10)$$

$$\text{s.t. } \sum_{j \in J} \sum_{l \in L} x_{ijlt} = 1 \quad \forall i \in I, \forall t \in T, \quad (11)$$

$$\sum_{i \in I} d_{it} x_{ijlt} \leq u_{jl} y_{jlt} \quad \forall j \in J, \forall l \in L, \forall t \in T, \quad (12)$$

$$y_{jlt} = y_{jlt(t-1)} + s_{jlt} + v_{jlt}^o - v_{jlt}^c \quad \forall j \in J, \forall l \in L, \forall t \in T, \quad (13)$$

$$\sum_{t'=1}^t v_{jlt'}^o \leq \sum_{t'=1}^t v_{jlt'}^c \quad \forall j \in J, \forall l \in L, \forall t \in T, \quad (14)$$

$$\sum_{l \in L} \sum_{t \in T} s_{jlt} \leq 1 \quad \forall j \in J, \quad (15)$$

$$x_{ijlt} \geq 0 \quad \forall i \in I, \forall j \in J, \forall l \in L, \forall t \in T, \quad (16)$$

$$s_{jlt}, v_{jlt}^o, v_{jlt}^c, y_{jlt} \in \{0, 1\} \quad \forall j \in J, \forall l \in L, \forall t \in T. \quad (17)$$

The objective function (10) minimizes the total costs composed by facility construction, maintenance of open facilities, and facility reopening and closing, as well as the costs to satisfy the customer demand. Constraints (11) are the demand constraints. Constraints (12) are the capacity constraints. The flow constraints (13) manage the state of a facility of a certain size, either open or closed. Constraints (14) ensure that a facility has to be temporarily closed before it can be reopened. Finally, constraints (15) state that at most one facility can be built at each location.

The SIs (8) can be adapted by replacing the right-hand side by y_{jlt} , and the ADCs (9) can be used by replacing the left-hand side by $\sum_{j \in J} \sum_{l \in L} u_{jl} y_{jlt}$.

4.1.2. Double Time Index Block Formulations.

Dias, Captivo, and Clímaco (2006) presented a linear MIP model that allows the repeated closing and

reopening of facilities. The model uses binary decision variables with two time indices, one for the opening and one for the closing of a facility. We extend this model by adding the choice of different facility capacity levels (note that we remove the constraints that require a minimum availability of open facilities). We also use a different notation to be consistent with our previously introduced notations. Binary variable $s_{jlt_1 t_2}$ is 1 if a facility of size l is constructed at location j at the beginning of time period t_1 and stays open until the end of period t_2 . Binary variable $y_{jlt_1 t_2}$ is 1 if an existing facility of size l , located at j , is reopened at the beginning of time period t_1 and stays open until the end of period t_2 . We let $\hat{f}_{jlt_1 t_2}^C$ denote the aggregated cost to construct a facility of size l at location j at time period t_1 , its maintenance costs from the beginning of period t_1 to the end of period t_2 , and the costs to temporarily close it at the end of period t_2 . We also let $\hat{f}_{jlt_1 t_2}^R$ denote the same type of cost for reopening an existing facility of size l instead of building it. These constants are computed as follows:

$$\hat{f}_{jlt_1 t_2}^C = f_{jl}^o + c_{jl}^c + (t_2 - t_1 + 1)F_{jl}^o \quad \text{and}$$

$$\hat{f}_{jlt_1 t_2}^R = c_{jl}^o + c_{jl}^c + (t_2 - t_1 + 1)F_{jl}^o.$$

Because the binary variables with two time indices describe capacity blocks through time, we refer to this formulation as the *double time index block formulation* (CR-2I)

$$(CR-2I) \min \left\{ \sum_{j \in J} \sum_{l \in L} \sum_{t_1 \in T} \sum_{t_2=t_1}^{|T|} (\hat{f}_{jlt_1 t_2}^C s_{jlt_1 t_2} + \hat{f}_{jlt_1 t_2}^R y_{jlt_1 t_2}) + \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} g_{ijlt} d_{it} x_{ijlt} \right\} \quad (18)$$

$$\text{s.t. } \sum_{j \in J} \sum_{l \in L} x_{ijlt} = 1 \quad \forall i \in I, \forall t \in T, \quad (19)$$

$$\sum_{t_2=t}^{|T|} y_{jlt_1 t_2} \leq \sum_{t_1=1}^{t-1} \sum_{t_2=t_1}^{t-1} s_{jlt_1 t_2} \quad \forall j \in J, \forall l \in L, \forall t \in T, \quad (20)$$

$$\sum_{l \in L} \sum_{t_1 \in T} \sum_{t_2=t_1}^{|T|} s_{jlt_1 t_2} \leq 1 \quad \forall j \in J, \quad (21)$$

$$\sum_{l \in L} \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} (s_{jlt_1 t_2} + y_{jlt_1 t_2}) \leq 1 \quad \forall j \in J, \forall t \in T, \quad (22)$$

$$\sum_{i \in I} d_{it} x_{ijlt} \leq \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} u_{jl} (s_{jlt_1 t_2} + y_{jlt_1 t_2}) \quad \forall j \in J, \forall l \in L, \forall t \in T, \quad (23)$$

$$x_{ijlt} \geq 0 \quad \forall i \in I, \forall j \in J, \quad \forall l \in L, \forall t \in T, \quad (24)$$

$$s_{jlt_1t_2}, y_{jlt_1t_2} \in \{0, 1\} \quad \forall j \in J, \forall l \in L, \quad (25)$$

$$\forall t_1 \in T, \forall t_2 \in T.$$

Constraints (19) are the demand constraints. Constraints (20) guarantee that a facility can only be reopened if it has been constructed and temporarily closed in an earlier period. Inequalities (21) impose that a facility can be constructed only once throughout the entire planning horizon. Constraints (22) guarantee that the intervals of open facilities (i.e., the capacity blocks) at the same location do not intersect. In other words, a facility can only be reopened if it is currently closed. In addition, these constraints also require that only one facility size l is selected at each location. Constraints (23) are the facility capacity constraints.

The SIs (8) can be adapted by replacing the right-hand side by $\sum_{t_1=1}^t \sum_{t_2=t}^{|T|} (s_{jlt_1t_2} + y_{jlt_1t_2})$. The ADCs (9) can be used by replacing the left-hand side with $\sum_{j \in J} \sum_{l \in L} \sum_{t_1=1}^t \sum_{t_2=t}^{|T|} u_{jl} (s_{jlt_1t_2} + y_{jlt_1t_2})$.

Strengthening the CR-2I formulation. Constraints (20) specify that, at each time period t , the capacity that is reopened at this period cannot be greater than the capacity that has been previously constructed. Consider the following LP relaxation solution scenario, where demands exist for three time periods t_1 , t_2 , and t_3 . A facility construction variable is selected with solution value 0.5, opening at the beginning of t_1 and closing at the end of t_1 (i.e., $s_{jlt_1t_1} = 0.5$). Facility reopening variables are then selected twice, each time with the same solution value of 0.5. The first reopening spans the time interval from the beginning of t_2 until the end of t_3 (i.e., $y_{jlt_2t_3} = 0.5$), whereas the second reopening spans the time interval from the beginning of t_3 until the end of t_3 (i.e., $y_{jlt_3t_3} = 0.5$). Separately, each of the last two reopenings is feasible in constraints (20). However, in total the solution reopens more capacity than has been made available through construction. To avoid such behavior in the LP relaxation solution, we may replace constraints (20) with the tighter set of constraints

$$\sum_{t_1=1}^t \sum_{t_2=t}^{|T|} y_{jlt_1t_2} \leq \sum_{t_1=1}^t \sum_{t_2=t_1}^{|T|} s_{jlt_1t_2} \quad \forall j \in J, \forall l \in L, \forall t \in T. \quad (26)$$

We denote the formulation given by (18), (19), and (21)–(26) as the CR-2I+ formulation.

4.1.3. Dominance Relationships. For any integer LP model P , let \bar{P} be the corresponding LP relaxation. For any model P , we denote by $v(P)$ its optimal value. For the three models presented for the DMCFLP_CR, the following relationships hold:

THEOREM 1. $v(\overline{CR-GMC}) = v(\overline{CR-1I}) \geq v(\overline{CR-2I})$.

PROOF. See Online Appendices A.1.1 (Theorem 4) and A.1.2 (Theorem 5) (available as supplemental material at <http://dx.doi.org/10.1287/trsc.2014.0575>). □

If constraints (20) in the CR-2I formulation are replaced by the strengthening constraints (26), all three formulations are equally strong:

THEOREM 2. $v(\overline{CR-GMC}) = v(\overline{CR-1I}) = v(\overline{CR-2I+})$.

PROOF. See Online Appendix A.1.3 (Theorems 4 and 7). □

4.2. Capacity Expansion and Reduction

We consider models for the facility location problem with capacity expansion and reduction, the DMCFLP_ER.

4.2.1. Single Time Index Flow Formulation. We modify the CR-1I as follows. Binary variables s_{jlt} now represent the total capacity expansion. A variable s_{jlt} is 1 if the capacity of the facility located at j is expanded by l capacity levels at the beginning of period t . Binary variable w_{jlt} is 1 if the capacity of a facility located at j is reduced by l capacity levels at the beginning of period t . We refer to this formulation as the *single time index flow formulation* (ER-1I)

$$(ER-1I) \quad \min \left\{ \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} (f_{jl}^o s_{jlt} + f_{jl}^c w_{jlt} + F_{jl}^o y_{jlt}) + \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} g_{ijlt} d_{it} x_{ijlt} \right\} \quad (27)$$

s.t. (11), (12)

$$\sum_{l \in L} l y_{jlt} = \sum_{l \in L} (l y_{jlt(t-1)} + l s_{jlt} - l w_{jlt}) \quad \forall j \in J, \forall t \in T, \quad (28)$$

$$\sum_{l \in L} y_{jlt} \leq 1 \quad \forall j \in J, \forall t \in T, \quad (29)$$

$$\sum_{l \in L} s_{jlt} \leq 1 \quad \forall j \in J, \forall t \in T, \quad (30)$$

$$\sum_{l \in L} w_{jlt} \leq 1 \quad \forall j \in J, \forall t \in T, \quad (31)$$

$$x_{ijlt} \geq 0 \quad \forall i \in I, \forall j \in J, \forall l \in L, \forall t \in T, \quad (32)$$

$$s_{jlt}, w_{jlt}, y_{jlt} \in \{0, 1\} \quad \forall j \in J, \forall l \in L, \forall t \in T. \quad (33)$$

The flow conservation constraints (28) manage the size of the facilities throughout the planning periods. Constraints (29)–(31), referred to as the *limiting constraints*, guarantee that the solution selects at most one capacity level for each type of variable y , s , and w . If the costs for facility maintenance, capacity expansion, and capacity reduction include economies of scale, these constraints are redundant, because the optimal

Downloaded from informs.org by [132.204.243.250] on 11 September 2017, at 10:39. For personal use only, all rights reserved.

solution will always choose a single capacity level: the one with the lowest cost in relation to its capacity.

The model may be seen as an adaptation of the relocation model of Wesolowsky and Truscott (1975), where capacity is expanded or reduced instead of relocated. It is also similar to the model presented by Jacobsen (1990) and to simplifications of the models presented by Melo, Nickel, and Saldanha-da-Gama (2006) and Behmardi and Lee (2008).

4.2.2. Double Time Index Block Formulations.

Dias, Captivo, and Clímaco (2007) allow multiple capacity blocks of different sizes at the same location. For each block, binary variables define the exact time interval during which the block is active. This accumulation of capacity blocks allows flexible capacity expansion and reduction, as previously discussed and exemplified in Figures 1(b) and 1(c). We extend this formulation to model the DMCFLP_ER.

Binary variables $y'_{jlt_1t_2}$ indicate whether a capacity block of size l is available at location j from the beginning of time period t_1 until the end of time period t_2 . Each capacity block may thus represent economies of scale in the function of its own size. However, in contrast to the ER-1I, the total capacity available at a location can now be composed of several capacity blocks. To consider economies of scale on the entire capacity involved at each location, we introduce additional binary variables y_{jlt} , which are 1 if the total capacity summed over all capacity blocks at location j available at time period t equals l . In the same manner, we introduce variables s_{jlt} and w_{jlt} to represent the total capacity that is added at a location (i.e., the construction of capacity blocks) or removed at a location (i.e., the closing of capacity blocks), respectively. Finally, as in the previous models, x_{ijlt} is the fraction of customer i 's demand that is served by a facility of size l at location j . The *double time index block formulation* (ER-2I) is given by

$$(ER-2I) \quad \min \left\{ \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} (f_{jl}^o s_{jlt} + f_{jl}^c w_{jlt} + F_{jl}^o y_{jlt}) + \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{t \in T} g_{ijlt} d_{it} x_{ijlt} \right\} \quad (34)$$

s.t. (11), (12), (29), (30), (31)

$$\sum_{l \in L} l s_{jlt} = \sum_{l \in L} \sum_{t_2=t} l y'_{jlt_1t_2} \quad \forall j \in J, \forall t \in T, \quad (35)$$

$$\sum_{l \in L} l w_{jlt} = \sum_{l \in L} \sum_{t_1=1}^{t-1} l y'_{jlt_1(t-1)} \quad \forall j \in J, \forall t \in T, \quad (36)$$

$$\sum_{l \in L} l y_{jlt} = \sum_{l \in L} \sum_{t_1=1}^t \sum_{t_2=t} l y'_{jlt_1t_2} \quad \forall j \in J, \forall t \in T, \quad (37)$$

$$x_{ijlt} \geq 0 \quad \forall i \in I, \forall j \in J, \quad \forall l \in L, \forall t \in T, \quad (38)$$

$$y'_{jlt_1t_2}, s_{jlt}, w_{jlt}, y_{jlt} \in \{0, 1\} \quad \forall j \in J, \quad \forall l \in L, \forall t_1 \in T, \forall t_2 \in T. \quad (39)$$

We adapt the demand and capacity constraints (11) and (12), respectively, from the previous models. Constraints (35)–(37) are the linking constraints that set the binary variables to benefit from economies of scale in the function of the total capacity involved in each operation and location. As for the ER-1I formulation, we also add the limiting constraints (29)–(31) as introduced in §4.2.1. The limiting constraints are necessary to ensure that feasible solutions use only one active variable of each type y , s , and w for each location and time period. These constraints have also proved to facilitate the solution process. We may also add the SIs and the ADCs.

4.2.3. Dominance Relationships. For the DMCFLP_ER, the ER-GMC formulation is stronger (strictly stronger for some instances) than the other two formulations:

$$\text{THEOREM 3. } v(\overline{ER-GMC}) \geq v(\overline{ER-1I}) = v(\overline{ER-2I}).$$

PROOF. See Online Appendices A.2.2 (Theorem 9) and A.2.1 (Theorem 10). \square

5. Computational Experiments

In this section, computational results are reported to illustrate the strength of the different formulations and their performance when using a state-of-the-art MIP solver to find optimal integer solutions. Computational experiments were performed for the two problem variants, DMCFLP_CR and DMCFLP_ER.

A large set of instances has been generated, varying a set of key parameters that were found to affect the difficulty of the problem. Instances have been generated with the following dimensions ($|J|/|I|$): (10/20), (10/50), (50/50), (50/100), (50/250), (100/250), (100/500), and (100/1,000). The highest capacity level at any facility, denoted by q , has been selected such that $q \in \{3, 5, 10\}$. Three different networks have been randomly generated on squares of the following sizes: 300 km, 380 km, and 450 km. We consider two different demand scenarios. In both scenarios, the demand for each of the customers is randomly generated and randomly distributed over time. The two scenarios differ in their total demand summed over all customers in each time period. In

the first scenario (*regular*), the total demand is similar in each time period. The second scenario (*irregular*) assumes that the total demand follows strong variations along time and therefore varies for each time period. Facility construction and operational costs follow concave cost functions; i.e., they involve economies of scale. All instances have also been generated with a second cost scenario in which the transportation costs are five times higher. Instances have been generated with $|T| = 12$, which may be interpreted as a planning horizon of one year divided into 12 months. This instance set contains a total of 288 instances. Note that we assume that the problem instances do not contain initially existing facilities. See Online Appendix B for a detailed description of the parameters used to generate the instances and Online Appendix C for details on the model sizes.

All mathematical models have been implemented in C/C++ using CPLEX 12.6. The code has been compiled and executed on openSUSE 11.3. Each problem instance has been run on a single Intel Xeon X5650 processor (2.67 GHz), limited to 24 GB of RAM.

5.1. Linear Relaxation Solution and Integrality Gaps

We compared the different formulations for the two problem variants by means of their LP relaxation bounds as well as the time necessary to solve the LP relaxations. All SIs have been added a priori. The ADCs were not added to these models, since they do not have any impact on the strength of the LP relaxation. For all instances, the LP relaxation was solved to optimality. Table 1 shows the average times to solve the LP relaxation, as well as the average integrality gaps, for each problem dimension and each number of maximum capacity levels q . The optimal integer solutions used to compute the integrality gaps were obtained by running CPLEX for up to 24 hours.

As previously shown, the CR-1I, the CR-2I+, and the CR-GMC formulations provide the same LP relaxation bound and thus the same integrality gap. However, the CR-GMC formulation solves the relaxation in slightly shorter computing times than the CR-1I and CR-2I+ formulations. For the DMCFLP_ER, the ER-1I and ER-2I formulations provide the same integrality gaps. Even though the computing times for the

Table 1 Average LP Relaxation Solution Time and Average Integrality Gaps for All Formulations

| q | Instance size | DMCFLP_CR | | | | DMCFLP_ER | | | | | |
|-----------|---------------|------------|---------|-------|-----------------|------------|-----------------|------------|-----------------|------------|-----------------|
| | | Time (sec) | | | Integr. gap (%) | ER-1I | | ER-2I | | ER-GMC | |
| | | 1I | 2I+ | GMC | | Time (sec) | Integr. gap (%) | Time (sec) | Integr. gap (%) | Time (sec) | Integr. gap (%) |
| 3 | 10/20 | 0.0 | 0.2 | 0.0 | 1.36 | 0.1 | 2.54 | 0.0 | 2.54 | 0.2 | 0.97 |
| | 10/50 | 0.3 | 0.3 | 0.1 | 0.33 | 0.2 | 0.96 | 0.0 | 0.96 | 0.3 | 0.34 |
| | 50/50 | 1.1 | 1.7 | 0.4 | 0.28 | 0.7 | 2.97 | 0.4 | 2.97 | 0.8 | 0.31 |
| | 50/100 | 1.7 | 2.6 | 0.8 | 0.01 | 0.7 | 1.34 | 0.7 | 1.34 | 0.6 | 0.03 |
| | 50/250 | 2.5 | 3.8 | 1.4 | 0.00 | 1.0 | 0.61 | 1.2 | 0.61 | 1.6 | 0.01 |
| | 100/250 | 8.3 | 10.3 | 4.4 | 0.02 | 3.8 | 0.99 | 4.4 | 0.99 | 5.3 | 0.02 |
| | 100/500 | 17.5 | 21.6 | 11.3 | 0.01 | 8.3 | 0.58 | 8.8 | 0.58 | 12.3 | 0.01 |
| | 100/1,000 | 34.3 | 51.4 | 28.4 | 0.01 | 19.7 | 0.37 | 23.3 | 0.37 | 28.4 | 0.00 |
| | Avg all | 8.2 | 11.5 | 5.9 | 0.25 | 4.3 | 1.29 | 4.8 | 1.29 | 6.2 | 0.21 |
| | 5 | 10/20 | 0.3 | 0.5 | 0.1 | 2.33 | 0.3 | 5.19 | 0.0 | 5.19 | 0.3 |
| 10/50 | | 0.6 | 1.0 | 0.4 | 0.80 | 0.2 | 2.08 | 0.1 | 2.08 | 0.7 | 0.68 |
| 50/50 | | 3.8 | 5.8 | 2.8 | 0.93 | 1.3 | 6.60 | 1.5 | 6.60 | 3.0 | 1.15 |
| 50/100 | | 6.5 | 8.4 | 2.8 | 0.18 | 1.9 | 2.79 | 2.4 | 2.79 | 3.5 | 0.19 |
| 50/250 | | 7.1 | 8.8 | 3.2 | 0.01 | 2.6 | 1.17 | 2.8 | 1.17 | 3.8 | 0.01 |
| 100/250 | | 26.6 | 24.3 | 10.8 | 0.03 | 8.3 | 1.93 | 9.4 | 1.93 | 15.1 | 0.03 |
| 100/500 | | 39.8 | 47.6 | 18.8 | 0.01 | 16.5 | 1.12 | 15.3 | 1.12 | 23.8 | 0.01 |
| 100/1,000 | | 74.6 | 85.5 | 43.6 | 0.01 | 33.4 | 0.70 | 46.9 | 0.70 | 49.8 | 0.00 |
| Avg all | | 19.9 | 22.7 | 10.3 | 0.54 | 8.1 | 2.70 | 9.8 | 2.70 | 12.5 | 0.49 |
| 10 | | 10/20 | 2.1 | 3.7 | 1.8 | 2.30 | 0.3 | 7.27 | 0.7 | 7.27 | 1.8 |
| | 10/50 | 3.6 | 7.1 | 4.1 | 1.38 | 0.8 | 4.81 | 1.1 | 4.81 | 3.4 | 0.70 |
| | 50/50 | 73.9 | 128.9 | 66.8 | 3.78 | 29.7 | 14.10 | 31.2 | 14.10 | 103.1 | 2.44 |
| | 50/100 | 125.3 | 207.7 | 96.2 | 1.25 | 38.9 | 7.15 | 45.0 | 7.15 | 101.1 | 1.07 |
| | 50/250 | 212.3 | 163.3 | 140.3 | 0.44 | 47.9 | 3.22 | 48.2 | 3.22 | 101.6 | 0.42 |
| | 100/250 | 1,126.1 | 1,011.4 | 940.0 | 0.53 | 274.2 | 4.73 | 285.7 | 4.73 | 829.2 | 0.47 |
| | 100/500 | 647.7 | 451.3 | 236.9 | 0.06 | 122.5 | 2.46 | 152.5 | 2.46 | 303.7 | 0.09 |
| | 100/1,000 | 325.6 | 421.3 | 138.9 | 0.01 | 116.6 | 1.47 | 140.0 | 1.47 | 158.2 | 0.01 |
| | Avg all | 190.8 | 184.0 | 120.8 | 1.10 | 50.3 | 5.85 | 54.8 | 5.85 | 125.7 | 0.78 |

ER-GMC formulation are higher than for the previous two formulations, the ER-GMC formulation provides a significantly smaller integrality gap.

5.2. CPLEX Optimization

Generic MIP solvers such as CPLEX incorporate several heuristics to find good-quality solutions early in the search tree and to improve the final solution quality. However, the use of such heuristics often leads to an unforeseeable behavior and does not allow for a proper comparison of different formulations for the same problem. We therefore compare the performance of the different formulations by considering two different optimization environments. The first one is a traditional branch-and-cut environment, which aims to test the formulations' ability to prove optimality. We used the MIP branch-and-cut algorithm of CPLEX 12.6 and turned off all heuristics (i.e., MIP heuristics, Feasibility Pump, Local Branching, and RINS). Instead, we used the solution value of the optimal integer solution as an artificial upper bound. This value is passed as a cut-off value in the branch-and-cut tree. In the second optimization scenario, we used CPLEX default settings; this reflects a typical use in practice.

For all experiments, computation times were limited to six hours. Furthermore, all SIs were added

a priori to the models. Even though the number of SIs may increase significantly, adding them a priori (instead of as *CPLEX user cuts* or even not at all) significantly facilitated the solution of the problems. Experiments showed that, for most of the problem instances, a large number of SIs were violated. CPLEX thus spends much time identifying and adding violated SIs when treated as *CPLEX user cuts*. Although redundant to the LP relaxation of the presented formulations, the ADCs tended to slightly facilitate the solution of the problems. Therefore, they also have been added to the formulations. For some models, the limiting constraints, as shown in §4.2, may not change the set of feasible integer solutions but still facilitate the solution of the problem. For example, for the ER-1I formulation, the average solution time for our test instances decreased by around 35%. The constraints are thus added to the models even if they are redundant.

5.2.1. Optimization in the Branch-and-Cut Environment. We now present computational results for the branch-and-cut environment. CPLEX offers three different search strategies (parameter *MIPsearch*): traditional branch and cut, dynamic search, and an automatic choice based on internal rules. Our experiments showed that the traditional branch-and-cut option

Table 2 CPLEX Branch-and-Cut Computing Times (in Seconds) for Instances Solved to Optimality by All Formulations for Each Problem

| q | Instance size | DMCFLP_CR | | | DMCFLP_ER | | | | |
|-----|---------------|-------------|-------|---------|-----------|-------------|-------|---------|--------|
| | | No. of inst | CR-1I | CR-2I+ | CR-GMC | No. of inst | ER-1I | ER-2I | ER-GMC |
| 3 | 10/20 | 12 | 0.9 | 11.3 | 1.2 | 12 | 0.3 | 1.4 | 0.3 |
| | 10/50 | 12 | 0.5 | 5.1 | 0.4 | 12 | 0.5 | 1.7 | 0.5 |
| | 50/50 | 11 | 6.3 | 7.8 | 2.3 | 12 | 421.8 | 1,015.9 | 161.4 |
| | 50/100 | 12 | 3.6 | 9.9 | 2.3 | 12 | 3.1 | 5.9 | 2.5 |
| | 50/250 | 12 | 3.4 | 23.2 | 3.8 | 12 | 6.0 | 9.6 | 5.2 |
| | 100/250 | 12 | 14.8 | 56.7 | 14.6 | 12 | 17.4 | 29.3 | 14.3 |
| | 100/500 | 12 | 28.2 | 127.0 | 31.3 | 12 | 34.7 | 65.7 | 29.4 |
| | 100/1,000 | 12 | 66.9 | 370.3 | 75.8 | 12 | 82.2 | 179.1 | 54.8 |
| | All | 95 | 15.7 | 77.1 | 16.6 | 96 | 70.7 | 163.6 | 33.6 |
| 5 | 10/20 | 12 | 13.7 | 470.4 | 11.8 | 12 | 7.8 | 199.8 | 4.3 |
| | 10/50 | 12 | 12.3 | 1,141.8 | 6.8 | 12 | 3.4 | 16.7 | 2.6 |
| | 50/50 | 9 | 13.9 | 18.9 | 4.3 | 8 | 24.4 | 139.9 | 7.8 |
| | 50/100 | 11 | 72.6 | 824.5 | 17.3 | 12 | 32.3 | 142.8 | 23.6 |
| | 50/250 | 12 | 8.8 | 46.8 | 9.8 | 12 | 11.4 | 21.0 | 10.1 |
| | 100/250 | 12 | 30.2 | 107.7 | 31.3 | 12 | 50.8 | 71.7 | 32.8 |
| | 100/500 | 12 | 45.9 | 230.2 | 47.8 | 12 | 83.6 | 131.4 | 56.4 |
| | 100/1,000 | 12 | 109.3 | 652.8 | 115.2 | 12 | 198.5 | 301.4 | 97.6 |
| | All | 92 | 38.8 | 446.0 | 31.5 | 92 | 52.7 | 127.6 | 30.3 |
| 10 | 10/20 | 8 | 529.1 | 899.4 | 159.5 | 3 | 26.0 | 8,762.3 | 5.3 |
| | 10/50 | 7 | 115.1 | 3,584.3 | 81.7 | 2 | 3.0 | 1,104.0 | 3.5 |
| | 50/50 | 4 | 85.5 | 37.3 | 16.0 | 1 | 53.0 | 2,387.0 | 19.0 |
| | 50/100 | 6 | 102.7 | 2,168.3 | 18.8 | 6 | 133.5 | 4,194.0 | 23.3 |
| | 50/250 | 8 | 243.9 | 1,830.3 | 101.0 | 6 | 47.8 | 203.0 | 36.0 |
| | 100/250 | 7 | 112.0 | 306.6 | 88.3 | 7 | 165.9 | 659.6 | 111.4 |
| | 100/500 | 11 | 198.8 | 931.0 | 165.5 | 11 | 531.6 | 2,131.2 | 284.7 |
| | 100/1,000 | 5 | 136.6 | 1,226.6 | 155.4 | 12 | 887.2 | 2,291.4 | 313.0 |
| | All | 56 | 207.2 | 1,403.7 | 108.0 | 48 | 393.3 | 2,350.4 | 168.0 |

Downloaded from informs.org by [132.204.243.250] on 11 September 2017, at 10:39. For personal use only, all rights reserved.

Table 3 CPLEX Branch-and-Cut Optimality Gaps for Instances of the DMCFLP_CR Not Solved Within 6 Hours

| q | Instance size | No. of inst | CR-1I | | | CR-2I+ | | | CR-GMC | | |
|-----|---------------|-------------|---------|------|--------|---------|------|--------|---------|------|--------|
| | | | Gap (%) | | No. ns | Gap (%) | | No. ns | Gap (%) | | No. ns |
| | | | Avg | Max | | Avg | Max | | Avg | Max | |
| 3 | 50/50 | 1 | 0.01 | 0.01 | 0 | 0.06 | 0.06 | 0 | 0.01 | 0.01 | 0 |
| 5 | 50/50 | 3 | 0.12 | 0.12 | 2 | — | — | 3 | 0.01 | 0.01 | 2 |
| | 50/100 | 1 | 0.01 | 0.01 | 0 | 0.02 | 0.02 | 0 | 0.01 | 0.01 | 0 |
| 10 | 10/20 | 4 | 0.05 | 0.13 | 1 | 0.63 | 0.63 | 3 | 0.12 | 0.31 | 0 |
| | 10/50 | 5 | 0.01 | 0.01 | 2 | 0.45 | 0.45 | 4 | 0.01 | 0.01 | 2 |
| | 50/50 | 8 | 0.10 | 0.10 | 7 | 0.01 | 0.01 | 7 | 0.01 | 0.01 | 6 |
| | 50/100 | 6 | 0.01 | 0.01 | 5 | — | — | 6 | 0.01 | 0.01 | 5 |
| | 50/250 | 4 | 0.01 | 0.01 | 3 | — | — | 4 | 0.00 | 0.01 | 2 |
| | 100/250 | 5 | 0.04 | 0.04 | 4 | — | — | 5 | 0.01 | 0.01 | 3 |
| | 100/500 | 1 | 0.01 | 0.01 | 0 | — | — | 1 | 0.01 | 0.01 | 0 |
| | 100/1,000 | 7 | 0.00 | 0.00 | 0 | — | — | 7 | 0.00 | 0.01 | 0 |
| | All | 40 | 0.02 | 0.13 | 22 | 0.37 | 0.63 | 37 | 0.03 | 0.31 | 18 |

performed slightly better than the other two options. All of the following results are therefore based on the traditional branch-and-cut scheme. Furthermore, all heuristics are turned off and the optimal integer solution value is passed to the solver as an upper bound cut-off value.

For each problem, the results have been separated into two groups: instances that have been solved to optimality by all formulations and instances where at least one formulation could not prove optimality within the given time limit. Table 2 summarizes the results for the instances that have been solved to optimality within the given time limit of six hours by all formulations for each problem. The table reports the number of instances that have been solved to optimality, as well as the average computation times to solve the instances for each of the formulations. For both problem variants, we observe that the 2I formulations perform the worst. Between the 1I and GMC-based formulations, the GMC-based models provide substantially better results.

Tables 3 and 4 summarize the results for instances where at least one of the formulations did not solve

the instances in the given time limit. The tables show average and maximum optimality gaps as reported by CPLEX, as well as the number of instances where the optimal solution has not been found within the given time limit (*No. ns*). Note that a positive optimality gap indicates that an optimal solution (i.e., the one with the cut-off value) has been found, but optimality has not been proven. For $q = 3$ and $q = 5$, a few instances with 50 facility locations have been found to be difficult to solve. All other instances are for $q = 10$. Again, the 2I formulations perform the worst, having the highest number of instances where the optimal solution has not been found. For both problem variants, the GMC finds more solutions than the 1I and 2I formulations. If the optimal solutions are found, the optimality gaps are low for all three formulations.

5.2.2. Optimization with CPLEX Default Settings. As shown in the previous section, the GMC-based formulation outperforms the 1I and 2I formulations for both problem variants in a traditional branch-and-cut environment, allowing a clear comparison of the formulations without the interference of heuristics. In practice, however, the objective is

Table 4 CPLEX Branch-and-Cut Optimality Gaps for Instances of the DMCFLP_ER Not Solved Within 6 Hours

| q | Instance size | No. of inst | ER-1I | | | ER-2I | | | ER-GMC | | |
|-----|---------------|-------------|---------|------|--------|---------|------|--------|---------|------|--------|
| | | | Gap (%) | | No. ns | Gap (%) | | No. ns | Gap (%) | | No. ns |
| | | | Avg | Max | | Avg | Max | | Avg | Max | |
| 5 | 50/50 | 4 | 0.01 | 0.01 | 3 | — | — | 4 | 0.05 | 0.12 | 1 |
| 10 | 10/20 | 9 | 0.01 | 0.01 | 0 | 0.37 | 0.37 | 8 | 0.01 | 0.01 | 0 |
| | 10/50 | 10 | 0.01 | 0.01 | 1 | — | — | 10 | 0.01 | 0.01 | 0 |
| | 50/50 | 11 | 0.01 | 0.01 | 8 | — | — | 11 | 0.00 | 0.01 | 7 |
| | 50/100 | 6 | 0.01 | 0.01 | 5 | — | — | 6 | 0.00 | 0.01 | 4 |
| | 50/250 | 6 | 0.01 | 0.01 | 2 | — | — | 6 | 0.03 | 0.12 | 1 |
| | 100/250 | 5 | 0.01 | 0.01 | 4 | — | — | 5 | 0.01 | 0.01 | 3 |
| | 100/500 | 1 | — | — | 1 | — | — | 1 | 0.01 | 0.01 | 0 |
| | All | 48 | 0.01 | 0.01 | 21 | 0.37 | 0.37 | 47 | 0.01 | 0.12 | 15 |

Table 5 Computing Times (in Seconds) Using CPLEX with Default Settings for Instances Solved to Optimality by All Formulations for Each Problem

| q | Instance size | DMCFLP_CR | | | | DMCFLP_ER | | | |
|-----|---------------|-------------|---------|---------|--------|-------------|---------|---------|--------|
| | | No. of inst | CR-1I | CR-2I+ | CR-GMC | No. of inst | ER-1I | ER-2I | ER-GMC |
| 3 | 10/20 | 12 | 1.1 | 5.7 | 1.5 | 12 | 0.3 | 1.4 | 0.3 |
| | 10/50 | 12 | 0.8 | 3.8 | 1.2 | 12 | 0.5 | 1.6 | 1.1 |
| | 50/50 | 12 | 121.8 | 158.4 | 18.3 | 12 | 302.4 | 1,402.6 | 116.2 |
| | 50/100 | 12 | 4.2 | 13.4 | 3.3 | 12 | 4.8 | 7.3 | 3.7 |
| | 50/250 | 12 | 4.3 | 25.3 | 5.6 | 12 | 7.5 | 12.3 | 6.8 |
| | 100/250 | 12 | 13.9 | 70.0 | 20.4 | 12 | 22.7 | 36.6 | 19.1 |
| | 100/500 | 12 | 36.5 | 155.0 | 36.3 | 12 | 45.9 | 75.5 | 36.8 |
| | 100/1,000 | 12 | 76.3 | 440.4 | 89.3 | 12 | 92.7 | 156.0 | 64.4 |
| All | 96 | 32.4 | 109.0 | 22.0 | 96 | 59.6 | 211.7 | 31.0 | |
| 5 | 10/20 | 12 | 10.2 | 43.0 | 10.4 | 12 | 7.3 | 42.3 | 5.8 |
| | 10/50 | 12 | 10.8 | 121.2 | 12.9 | 12 | 5.0 | 25.1 | 5.0 |
| | 50/50 | 10 | 194.6 | 176.1 | 62.0 | 9 | 663.0 | 2,126.3 | 84.2 |
| | 50/100 | 12 | 447.9 | 518.8 | 143.3 | 12 | 84.6 | 161.8 | 35.3 |
| | 50/250 | 12 | 10.2 | 51.8 | 11.7 | 12 | 14.8 | 29.2 | 13.8 |
| | 100/250 | 12 | 40.3 | 136.5 | 41.1 | 12 | 61.1 | 104.4 | 46.0 |
| | 100/500 | 12 | 65.3 | 270.9 | 56.1 | 12 | 119.5 | 160.3 | 69.5 |
| | 100/1,000 | 12 | 128.1 | 741.3 | 143.4 | 12 | 192.8 | 331.8 | 126.8 |
| All | 94 | 111.7 | 259.2 | 60.1 | 93 | 126.8 | 316.1 | 47.1 | |
| 10 | 10/20 | 8 | 59.8 | 903.8 | 52.9 | 8 | 55.0 | 2,808.0 | 10.9 |
| | 10/50 | 7 | 119.3 | 1,033.6 | 108.3 | 8 | 180.9 | 4,310.8 | 28.6 |
| | 50/50 | 5 | 184.4 | 61.6 | 44.4 | 5 | 392.0 | 2,946.4 | 67.0 |
| | 50/100 | 7 | 744.1 | 1,595.0 | 97.4 | 7 | 577.0 | 3,186.6 | 162.1 |
| | 50/250 | 10 | 1,824.1 | 2,018.3 | 289.4 | 9 | 1,747.9 | 4,865.4 | 257.2 |
| | 100/250 | 8 | 2,009.1 | 1,049.3 | 503.8 | 7 | 258.3 | 963.0 | 125.6 |
| | 100/500 | 11 | 208.0 | 701.5 | 215.3 | 11 | 806.2 | 3,565.6 | 416.9 |
| | 100/1,000 | 8 | 420.3 | 1,760.5 | 355.8 | 12 | 957.1 | 2,809.6 | 389.8 |
| All | 64 | 740.8 | 1,192.4 | 222.2 | 67 | 683.3 | 3,245.6 | 212.6 | |

most often to find high-quality solutions in short computing times. CPLEX incorporates several heuristics to find good-quality solutions early in the search tree. We now compare the different formulations using CPLEX with default settings, making full use of the heuristic capabilities of the MIP solver.

Computational experiments on the same set of test instances indicate trends similar to those observed in the experiments of §5.2.1. The results for the instances that have been solved by all formulations for each problem are summarized in Table 5. The table reports

the number of instances that have been solved to optimality, as well as the average computing times to solve the instances for each of the formulations. As in the previous experiments, the 2I formulation performs the worst. Between the 1I and the GMC-based formulations, the GMC-based models are solved in substantially shorter computing times.

Tables 6 and 7 summarize the results for instances where at least one of the formulations did not solve the instances in the time limit. The tables show average and maximum optimality gaps as reported by CPLEX,

Table 6 Optimality Gaps Using CPLEX with Default Settings for Instances of the DMCFLP_CR Not Solved Within 6 Hours

| q | Instance size | No. of inst | CR-1I | | | CR-2I+ | | | CR-GMC | | |
|-----|---------------|-------------|---------|------|--------|---------|------|--------|---------|------|--------|
| | | | Gap (%) | | | Gap (%) | | | Gap (%) | | |
| | | | Avg | Max | No. ns | Avg | Max | No. ns | Avg | Max | No. ns |
| 5 | 50/50 | 2 | 0.99 | 1.18 | 0 | 1.17 | 1.32 | 0 | 0.18 | 0.35 | 0 |
| 10 | 10/20 | 4 | 0.01 | 0.01 | 0 | 0.72 | 0.96 | 0 | 0.01 | 0.01 | 0 |
| | 10/50 | 5 | 0.12 | 0.56 | 0 | 0.56 | 1.36 | 0 | 0.26 | 0.87 | 0 |
| | 50/50 | 7 | 1.85 | 3.73 | 0 | 1.46 | 4.21 | 0 | 1.36 | 3.42 | 0 |
| | 50/100 | 5 | 1.14 | 2.54 | 0 | 0.87 | 1.84 | 0 | 0.58 | 1.43 | 0 |
| | 50/250 | 2 | 0.59 | 0.85 | 0 | 0.59 | 0.89 | 0 | 0.42 | 0.75 | 0 |
| | 100/250 | 4 | 1.10 | 2.76 | 0 | 0.67 | 1.61 | 0 | 0.69 | 1.69 | 0 |
| | 100/500 | 1 | 0.01 | 0.01 | 0 | 0.04 | 0.04 | 0 | 0.01 | 0.01 | 0 |
| | 100/1,000 | 4 | 0.00 | 0.00 | 0 | — | — | 4 | 0.00 | 0.01 | 0 |
| All | 32 | 0.78 | 3.73 | 0 | 0.86 | 4.21 | 4 | 0.54 | 3.42 | 0 | |

Table 7 Optimality Gaps Using CPLEX with Default Settings for Instances of the DMCFLP_ER Not Solved Within 6 Hours

| q | Instance size | No. of inst | ER-1I | | | ER-2I | | | ER-GMC | | |
|----|---------------|-------------|---------|------|--------|---------|------|--------|---------|------|--------|
| | | | Gap (%) | | No. ns | Gap (%) | | No. ns | Gap (%) | | No. ns |
| | | | Avg | Max | | Avg | Max | | Avg | Max | |
| 5 | 50/50 | 3 | 0.50 | 1.00 | 0 | 1.01 | 1.33 | 0 | 0.00 | 0.00 | 0 |
| 10 | 10/20 | 4 | 0.01 | 0.01 | 0 | 1.60 | 2.76 | 0 | 0.01 | 0.01 | 0 |
| | 10/50 | 4 | 0.01 | 0.01 | 0 | 1.22 | 1.65 | 0 | 0.01 | 0.01 | 0 |
| | 50/50 | 7 | 1.43 | 3.23 | 0 | 3.12 | 5.08 | 1 | 0.55 | 1.30 | 0 |
| | 50/100 | 5 | 0.83 | 1.47 | 0 | 1.47 | 2.45 | 0 | 0.45 | 1.09 | 0 |
| | 50/250 | 3 | 0.32 | 0.74 | 0 | 0.60 | 1.06 | 0 | 0.12 | 0.35 | 0 |
| | 100/250 | 5 | 0.52 | 1.22 | 0 | 2.05 | 6.85 | 0 | 0.34 | 1.11 | 0 |
| | 100/500 | 1 | 0.12 | 0.12 | 0 | 0.55 | 0.55 | 0 | 0.01 | 0.01 | 0 |
| | All | 29 | 0.62 | 3.23 | 0 | 1.78 | 6.85 | 1 | 0.29 | 1.30 | 0 |

as well as the number of instances where no feasible solution has been found (*No. ns*). For $q = 5$, the few instances that have been found to be difficult to solve are those with 50 facility locations. All other instances are for $q = 10$. Again, the 2I formulations perform the worst. For some of the instances, the formulation did not find any feasible solution. The GMC formulation performs similarly to the 1I formulation for the DMCFLP_CR and presents slightly better results than the 1I formulation for the DMCFLP_ER.

5.3. Closing and Reopening with Capacity Expansion and Reduction

The two problem variants consider either facility closing/reopening or capacity expansion/reduction. Experiments have also been performed for a third problem variant combining both features, referred to as the DMCFLP_CRER. The problem is modeled using the DFLPG by using the transition arcs for both problems, as shown in §3.2. Additionally, arcs are added that represent combined decisions such as facility reopening with subsequent capacity expansion (in the same time period) and capacity reduction with subsequent facility closing. Alternatively, a specialized flow

formulation can be used with two types of flow constraints: one to manage the capacity of open facilities and one to manage the capacity of closed facilities. The advantage of the GMC model for this variant is even more obvious than what was observed for the DMCFLP_ER. We proved that the GMC-based model provides a stronger LP relaxation than the specialized flow formulation. Computationally, the average integrality gap (for all instances with $q = 10$) improved from 6.00% to 1.06% when using the GMC-based model instead of the specialized formulation. In the traditional branch-and-cut environment, using CPLEX without heuristics and providing it with the optimal integer solution value as a cut-off, the flow formulation takes on average 1,820 seconds to solve the instances of size $q = 10$, whereas the GMC-based formulation solves the same instances in an average time of only 206 seconds, about nine times faster. Using CPLEX default settings, the dominance of the GMC-based formulation is mainly preserved. The average computation time improves from 1,924 to 313 seconds.

5.4. Solution Structure and Instance Properties

We now analyze the structure of the optimal or near-optimal solutions. Figure 2 illustrates for each

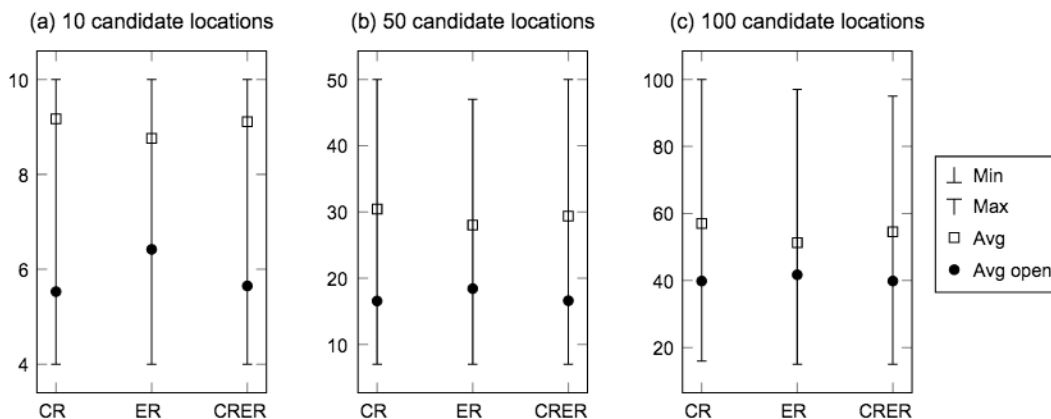


Figure 2 Structure of Optimal Solutions: Minimum, Average, and Maximum Number of Selected Facility Locations, as well as the Average Number of Open Facilities per Time Period Throughout the Entire Planning Horizon

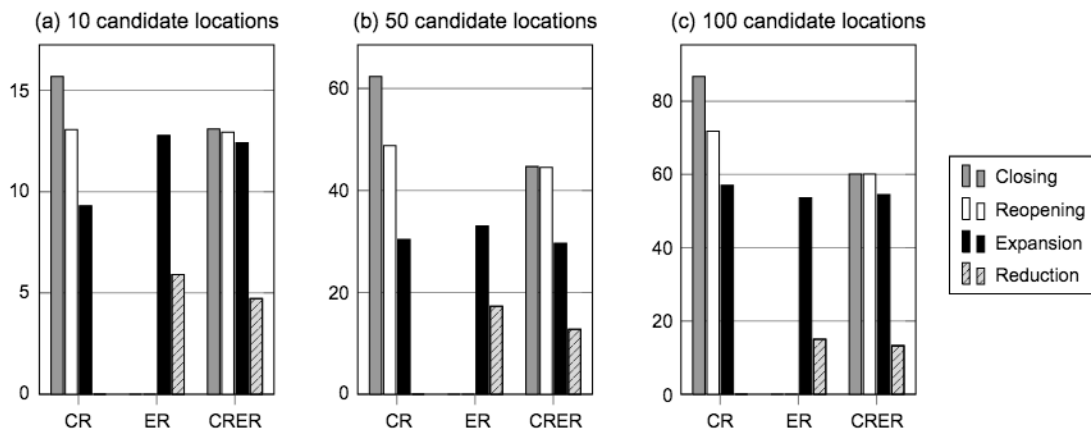


Figure 3 Structure of Optimal Solutions: Average Number of Facility Closings and Reopenings, as well as Capacity Reductions and Expansions

problem variant and problem size (10, 50, and 100 candidate facility locations) the minimum (*Min*), maximum (*Max*), and average (*Avg*) number of selected facility locations. Since a facility may not be available in all of the subsequent time periods after its construction, a second average value (*Avg open*) indicates the average number of facilities that are available (i.e., having $l \geq 1$) at each time period. The results are surprisingly similar for the three problem variants CR, ER, and CRER. On average, about half of the candidate locations have been selected. These facilities are active only in about two-thirds of the planning horizon. For the CR, this is done by closing a facility. For the ER, the capacity is reduced to level 0.

Figure 3 shows different indicators of the solutions structure: the average number of facility closings and reopenings, as well as the average number of capacity expansions and reductions. It can be observed that the average values for certain indicators such as capacity expansion and reduction are similar for the three problem variants. Based on these results, one may conclude that the main driver to adjust capacities are high maintenance costs and therefore that high-quality solutions tend to provide a total capacity that only slightly exceeds the total demand.

However, an analysis of the solutions for smaller instances reveals that the selected opening schedules are very different for the three problem variants when the original transportation costs are used. In contrast, the opening schedules are very similar when the transportation costs are set five times higher. Table 8 presents the impact of these instance properties on the solution structure. The table shows, for each of the indicators, the average number of occurrences in instances with the original transportation costs and in instances where the transportation costs are five times higher. In the same way, it indicates the number of occurrences in instances with regular and irregular demand distribution. The impact of these

instance properties has been found to be very similar for all three problem variants and is here exemplified for the DMCFLP_CRER, showing the average values over all instances. We can identify a clear trend. Solutions for instances with original transportation costs involve only a few operations that adjust the capacities throughout the planning horizon and therefore tend to serve the demand from a similar set of facility locations. Solutions for instances with high transportation costs provide capacities that tend to geographically follow the demand along time, constructing on average more than twice the number of facilities and performing two to three times the operations that adjust capacities along time. In both cases the maintenance costs are the same, so the motivating factor to geographically shift capacity is given by high transportation costs and the effort to bring capacities closer to the demand. Regarding the demand distribution, an irregular demand distribution results in only slightly more capacity adjustments than a regular demand distribution.

Impact on problem difficulty. The instance characteristics not only impact the solution structure but also the difficulty of solving the problem. The computing time for instances with irregular total customer demand is, on average, 30% lower than for instances where the total customer demand is regular at each

Table 8 Impact of Instance Characteristics (Transportation Costs and Demand Distribution) on the Solution Structure for the DMCFLP_CRER

| # | Transportation costs | | Demand distribution | |
|----------------------|----------------------|------------|---------------------|-----------|
| | original | 5 × higher | regular | irregular |
| Constructions | 21.7 | 44.9 | 33.3 | 33.3 |
| Closings | 21.0 | 63.3 | 40.3 | 43.9 |
| Reopenings | 21.0 | 63.2 | 40.3 | 43.8 |
| Capacity expansions | 22.3 | 46.3 | 33.8 | 34.7 |
| Capacity reductions | 5.3 | 16.4 | 10.6 | 11.1 |
| Avg. open facilities | 16.2 | 28.4 | 23.2 | 21.3 |

Table 9 Impact of Number of Time Periods in Problem Instances ($q = 10$) for the CRER-GMC Formulation when Using CPLEX with Default Settings

| Instance size | T = 6 | | T = 8 | | T = 10 | | T = 12 | | T = 14 | |
|---------------|---------|------------|---------|------------|---------|------------|---------|------------|---------|------------|
| | Gap (%) | Time (sec) | Gap (%) | Time (sec) | Gap (%) | Time (sec) | Gap (%) | Time (sec) | Gap (%) | Time (sec) |
| 10/20 | 0.00 | 112.1 | 0.01 | 180.8 | 0.01 | 140.0 | 0.01 | 940.7 | 0.01 | 2,569.4 |
| 10/50 | 0.00 | 73.0 | 0.01 | 129.1 | 0.01 | 302.4 | 0.01 | 1,822.0 | 0.06 | 6,203.6 |
| 50/50 | 0.17 | 7,842.3 | 0.25 | 8,818.4 | 0.45 | 10,820.0 | 1.23 | 10,913.3 | 0.96 | 12,800.9 |
| 50/100 | 0.02 | 2,126.8 | 0.11 | 4,107.6 | 0.17 | 4,201.6 | 0.56 | 7,582.8 | 0.40 | 9,225.2 |
| 50/250 | 0.01 | 446.0 | 0.02 | 2,002.3 | 0.01 | 1,945.0 | 0.14 | 7,304.1 | 0.12 | 5,655.8 |
| 100/250 | 0.05 | 2,883.7 | 0.08 | 5,516.9 | 0.14 | 6,940.4 | 0.57 | 8,899.3 | 0.31 | 9,481.0 |
| 100/500 | 0.00 | 339.3 | 0.00 | 988.9 | 0.00 | 940.8 | 0.01 | 2,687.5 | 0.01 | 2,571.3 |
| 100/1,000 | 0.00 | 414.5 | 0.00 | 463.9 | 0.00 | 538.4 | 0.00 | 690.2 | 0.00 | 549.1 |
| All | 0.03 | 1,779.7 | 0.06 | 2,776.0 | 0.10 | 3,228.6 | 0.19 | 4,201.8 | 0.23 | 6,132.0 |

time period. In contrast, the ratio between transportation and facility construction costs has a much larger impact. Instances where the transportation costs are five times higher than the original costs are, on average, solved around 60 times faster. This is directly linked to the integrality gap for those instances, which is significantly lower if the ratio between facility construction and transportation costs is not close to 1.

Finally, we also analyzed the impact of the length of the planning horizon in the problem instances, using CPLEX with its default settings. For this purpose, instances have also been generated with different numbers of time periods such that $|T| \in \{6, 8, 10, 12, 14\}$. Table 9 summarizes the average computation times and average optimality gaps for the CRER-GMC formulation. The computational results are presented for five different numbers of time periods $|T|$: 6, 8, 10, 12, and 14. The results are very consistent, showing that the difficulty of the problems increases proportionally to the number of time periods. For the CRER-II formulation, a similar trend was observed. However, the CRER-II was clearly outperformed by the CRER-GMC for all tested lengths of the planning horizon.

6. Conclusions and Future Research

We have introduced a new general facility location problem that unifies several existing multiperiod facility location problems. We showed the flexibility of this generalization by focusing on two problem variants: facility closing and reopening and capacity expansion and reduction. In addition, we also reported results on a variant that combines both of these features. For the first two cases, we derived specialized models based on two well known formulation approaches. We formally proved that, even though our model is more general, it provides LP relaxation bounds as strong as the other formulations for the case of facility closing/reopening and stronger LP relaxation bounds than the formulations for the other two cases. Computational experiments showed

that, for the two variants involving capacity expansion and reduction, the integrality gap of our model is up to seven times smaller than the integrality gaps of the specialized formulations. When assessing the performance of the models in a traditional branch-and-cut environment, the GMC-based models solved the instances, on average, up to nine times faster than the specialized formulations. Using CPLEX default settings to solve the problem, the GMC-based models are, on average, up to six times faster.

The general model may also be used to model other problem variants not addressed in this work, e.g., the closing and reopening model of Chardaire, Sutter, and Costa (1996) or the dynamic location problem of Sridharan (1995). In addition, problem variants that involve capacity changes may benefit from the proposed modeling technique to strengthen the existing models. Problems such as those presented by Shulman (1991) and Correia and Captivo (2003) can be modeled by the DFLPG when adding individual constraints such as minimum production bounds for the facilities. Finally, as the general model is already very strong, it may also be an ideal candidate for decomposition techniques such as Lagrangian relaxation to find good-quality solutions in short computation times.

Supplemental Material

Supplemental material to this paper is available at <http://dx.doi.org/10.1287/trsc.2014.0575>.

Acknowledgments

The authors are grateful to Mitacs, the Natural Sciences and Engineering Research Council of Canada (NSERC) [Grant 2014-04959], and the Fonds de recherche du Québec Nature et Technologies for their financial support. The authors also thank three anonymous referees for their valuable comments, which have helped improve the paper.

References

- Arabani AB, Farahani RZ (2011) Facility location dynamics: An overview of classifications and applications. *Comput. Indust. Engrg.* 62(1):408–420.

- Behmardi B, Lee S (2008) Dynamic multi-commodity capacitated facility location problem in supply chain. Fowler J, Mason S, eds. *Proc. 2008 Indust. Engrg. Res. Conf.*, 1914–1919.
- Brotcorne L, Laporte G, Semet F (2003) Ambulance location and relocation models. *Eur. J. Oper. Res.* 147(3):451–463.
- Canel C, Khumawala BM, Law J, Loh A (2001) An algorithm for the capacitated, multi-commodity multi-period facility location problem. *Comput. Oper. Res.* 28(5):411–427.
- Chardaire P, Sutter A, Costa MC (1996) Solving the dynamic facility location problem. *Networks* 28(2):117–124.
- Correia I, Captivo ME (2003) A Lagrangian heuristic for a modular capacitated location problem. *Ann. Oper. Res.* 122(1–4):141–161.
- Correia I, Gouveia L, Saldanha-da-Gama F (2010) Discretized formulations for capacitated location problems with modular distribution costs. *Eur. J. Oper. Res.* 204(2):237–244.
- Daskin MS (2008) What you should know about location modeling. *Naval Res. Logist.* 55(4):283–294.
- Dias J, Captivo ME, Clímaco J (2006) Capacitated dynamic location problems with opening, closure and reopening of facilities. *IMA J. Management Math.* 17(4):317–348.
- Dias J, Captivo ME, Clímaco J (2007) Dynamic location problems with discrete expansion and reduction sizes of available capacities. *Investigação Operacional* 27(2):107–130.
- Farahani RZ, Abedian M, Sharahi S (2009) Dynamic facility location problem. Farahani RZ, Hekmatfar M, eds. *Facility Location—Concepts, Models, Algorithms and Case Studies*, Contributions to Management Sci. (Physica-Verlag HD, Heidelberg), 347–372.
- Gendron B, Crainic TG (1994) Relaxations for multicommodity capacitated network design problems. Technical report, Publication CRT-945, Centre de recherche sur les transports, Université de Montréal, Montréal.
- Hamacher H, Nickel S (1998) Classification of location models. *Location Sci.* 6(1–4):229–242.
- Harkness J (2003) Facility location with increasing production costs. *Eur. J. Oper. Res.* 145(1):1–13.
- Hinojosa Y, Kalcsics J, Nickel S, Puerto J, Velten S (2008) Dynamic supply chain design with inventory. *Comput. Oper. Res.* 35(2):373–391.
- Jacobsen SK (1990) Multiperiod capacitated location models. Mirchandani PB, Francis RL, eds. *Discrete Location Theory* (John Wiley & Sons, New York), 173–208.
- Jena SD, Cordeau JF, Gendron B (2012) Modeling and solving a logging camp location problem. *Ann. Oper. Res.*, ePub ahead of print December 28, <http://dx.doi.org/10.1007/s10479-012-1278-z>.
- Klose A, Drexl A (2005) Facility location models for distribution system design. *Eur. J. Oper. Res.* 162(1):4–29.
- Luss H (1982) Operations research and capacity expansion problems: A survey. *Oper. Res.* 30(5):907–947.
- Melkote S, Daskin MS (2001) Capacitated facility location/network design problems. *Eur. J. Oper. Res.* 129(3):481–495.
- Melo MT, Nickel S, Saldanha-da-Gama F (2006) Dynamic multi-commodity capacitated facility location: A mathematical modeling framework for strategic supply chain planning. *Comput. Oper. Res.* 33(1):181–208.
- Melo MT, Nickel S, Saldanha-da-Gama F (2009) Facility location and supply chain management: A review. *Eur. J. Oper. Res.* 196(2):401–412.
- Owen SH, Daskin MS (1998) Strategic facility location: A review. *Eur. J. Oper. Res.* 111(3):423–447.
- Peeters D, Antunes AP (2001) On solving complex multi-period location models using simulated annealing. *Eur. J. Oper. Res.* 130(1):190–201.
- Shulman A (1991) An algorithm for solving dynamic capacitated plant location problems with discrete expansion sizes. *Oper. Res.* 39(3):423–436.
- Sridharan R (1995) The capacitated plant location problem. *Eur. J. Oper. Res.* 87(2):203–213.
- Troncoso J, Garrido R (2005) Forestry production and logistics planning: An analysis using mixed-integer programming. *Forest Policy Econom.* 7(4):625–633.
- Van Roy TJ, Erlenkotter D (1982) A dual-based procedure for dynamic facility location. *Management Sci.* 28(10):1091–1105.
- Wesolowsky GO, Truscott WG (1975) The multiperiod location-allocation problem with relocation of facilities. *Management Sci.* 22(1):57–65.
- Wu L, Zhang X, Zhang J (2006) Capacitated facility location problem with general setup cost. *Comput. Oper. Res.* 33(5):1226–1241.